Appendix A

Variables & Assumptions

1. Self-judgment $S$ is regressed on true score $T$ to create $\hat{S}$

2. Residual score $Rs = S - \hat{S}$.

3. By definition, $r_{r,t} = 0$ and $r_{\hat{S},T} = 0$.

4. The derivation of interest is the correlation (Pearson’s $r$) between $S$ and $Rs$:

   
   \[
   r_{S,Rs} = \frac{\text{cov}(S, Rs)}{\sqrt{\text{var}(S) \times \text{var}(Rs)}}
   \]

5. The variance of $S$, $\text{var}(S)$, can be represented as $\text{cov}(S, S)$.

6. The correlation of interest, $r(S, Rs)$ can be represented as $r(\hat{S} + Rs, Rs)$, because $\hat{S} + Rs$ is equal to $S$.

Proof

\[
\begin{align*}
    r_{S,Rs} &= \frac{\text{cov}(\hat{S} + Rs, Rs)}{\sqrt{\text{var}(S) \times \text{var}(Rs)}} \\
    r_{S,Rs} &= \frac{\text{cov}(\hat{S}, Rs) + \text{cov}(Rs, Rs)}{\sqrt{\text{var}(S) \times \text{var}(Rs)}} \\
    r_{S,Rs} &= \frac{0 + \text{var}(Rs)}{\sqrt{\text{var}(S) \times \text{var}(Rs)}} \\
    \end{align*}
\]

(Note: Square var(Rs) in numerator here to get two separate $\sqrt{\text{var}(Rs)}$)

\[
\begin{align*}
    r_{S,Rs} &= \frac{0 + \sqrt{\text{var}(Rs)} \times \sqrt{\text{var}(Rs)}}{\sqrt{\text{var}(Ss) \times \sqrt{\text{var}(Rs)}}} \\
    r_{S,Rs} &= \frac{\sqrt{\text{var}(Rs)}}{\sqrt{\text{var}(S)}} \\
    r_{S,Rs} &= \frac{\text{SD}(Rs)}{\text{SD}(S)}
    \end{align*}
\]